(Result 1) For any sentences $P_1, \ldots, P_n$ and Q in M, if Q is a K-semantic consequence of $P_1, \ldots, P_n$ then Q is a logical consequence of $P_1, \ldots, P_n$.

Argument (relying on Necesary Bivalence).

(Necessary Bivalence) Necessarily, every sentence meaning is either true or false.

In order to argue for (Result 1), I will first establish the following lemma.

(Lemma) For any meaning interpretation m of M, for any possible world u, there is a K-model $\langle W, R, v \rangle$ and a $w \in W$ such that for any sentence A in M, A is true at w under v iff A is true at u under m

Proof of (Lemma). Let m be a meaning interpretation of M, and let u be a possible world. Let W be the set of possible worlds, R be the set

$\{ \langle w, w' \rangle \mid w'$ is possible relative to $w \}$, and

let v be the function on the set of sentential constants such that, for any sentence constant p in M, for any $w \in W$,

$v_w(p) = 1$ ~~iff~~ if $m(p)$ is true at w, and $v_w(p) = 0$ if $m(p)$ is false at w. (Given Necessary Bivalence, this defines v.)

We now need to establish (*) is true for every sentence A in M.

(*) A is true at u under v iff A is true at u under m

The proof of (*) is by induction on the length of A.

Base case It follows from the above definitions that, for each sentential constant p in M, p is true at n under v iff p is true at n under m

Inductive case We need to prove that, if A and B satisfy ~~the~~ (*), then $\neg A$, $A \wedge B$, $(A \vee B)$, $(A \supset B)$, $(A \equiv B)$, $\Box A$ and $\Diamond A$ do as well.

I will do the cases of conjunction and necessity. The other cases are similar.

The case of conjunction Suppose A and B satisfy (*). Then:

(i) A is true at n under v iff A is true at n under m

(ii) B is true at n under v iff B is true at n under m

By the definition of the extension of v to all sentences in M,

iii) '$A \wedge B$' is true at n under v iff A is true at n under v and B is true at n under v

Fourth, it follows from the meaning of 'and' that

iv) '$A \wedge B$' is true at u under m iff

A is true at u under m and B is true at u under m

It then follows from (i–iv) that

'$A \wedge B$' is true at u under v iff '$A \wedge B$' is true at u under m

Hence '$A \wedge B$' satisfies (*).

The case of necessity

Since A satisfies (*), we have

(i) A is true ~~at~~ at u under v iff

A is true at u under m

By the definition of the extension of v to all sentences in M,

ii) '$\Box A$' is true at u under v iff,

for any $w \in W$ st $\langle u, w \rangle \in R$, A is true at w under v

It follows from the possible worlds analysis of necessity that

iii) '$\Box A$' is true at u under m iff, for any pos world w that is possible relative to u, A is true at w under m

It now follows from (i–iii), ~~that~~ and the definitions of W and R that '$\Box A$' is true at u under v iff '$\Box A$' is true at u under m

Hence '$\Box A$' satisfies (*).

We have now established (*). ~~It~~ ~~follows~~

Since (Lemma ‡) follows from (*), we have established (Lemma ‡).

$\Box$

(Lemma) can now be used to establish (Result))
as follows.

Suppose Q is not a logical consequence of $P_1, \dots, P_n$. Then there is a meaning interpretation m ~~such~~ such that $P_1, \dots, P_n$ are all true under m, but Q is false under m. Letting $\underline{u}$ be the ~~act~~ actual world, it follows that $P_1, \dots, P_n$ are all true ~~un~~ at $\underline{u}$ under m, but Q is false ~~under~~ at $\underline{u}$ under m. By (Lemma), it then follows that there is a K-model $\langle W, R, v \rangle$ and a $w \in W$ such that $P_1, \dots, P_n$ are all true (at w) under v, but Q is false at w under v. Hence, Q is not a K-semantic consequence of $P_1, \dots, P_n$.

We have now established that if Q is not a logical consequence of $P_1, \ldots, P_n$ then Q is not a K-semantic consequence of $P_1, \ldots, P_n$. (Result 1) follows from this.